Convergence of Magnus integral addition theorems for confluent hypergeometric functions

HOWARD S. COHL[*], HANS VOLKMER[†], JESSICA E. HIRTENSTEIN[‡]

[*]Applied and Computational Mathematics Division, National Institute of Standards and Technology, Gaithersburg, MD, U.S.A.

[†]Department of Mathematical Sciences, University of Wisconsin-Milwaukee, Milwaukee, WI, U.S.A.

[‡]Department of Physics, University of California, Davis, CA, U.S.A.

Email: [*]hcohl@nist.gov, [†]volkmer@uwm.edu, [‡]jessie.hirtenstein@gmail.com

In 1946, Magnus presented an addition theorem for the confluent hypergeometric function of the second kind U with argument $x + y$ expressed as an integral of a product of two U's, one with argument x and another with argument y. We take advantage of recently obtained asymptotics for U with large complex first parameter by Volkmer (SIGMA **12** (2016), 046) to determine a domain of convergence for Magnus' result. Using well-known specializations of U, we obtain corresponding integral addition theorems with precise domains of convergence for modified parabolic cylinder functions, and Hankel, Macdonald, and Bessel functions of the first and second kind with order zero and one.